Module 8 · Portfolio theory and practice P14 · 8.1–8.3

SM pp. 299–359 (exercise 360–382). **2–11 marks a paper** (expected 6.9): **Q5(b) portfolio risk** in D23, D24, J25 (J24 Q5(a)) — **D23 = J25 = SM 8.3 Illustration 15 with new numbers** (portfolio beta, residual variance, Sharpe single-index v Markowitz variance); J26 Q5(b) holding-period / expected return; **MCQ (vii)** on utility / risk penalty / CML five times.

Paper	Long	MCQ
Jun'23	Q5(a) portfolio as risky as the market: amounts in stock and debentures	–
Dec'23	**Q5(b) portfolio β, residual variance, Sharpe index and Markowitz variance**	risk penalty = variance ÷ tolerance
Jun'24	Q5(a) betas, covariances, total and systematic risk, equal weights	utility = return − variance ÷ tolerance
Dec'24	Q5(b) correlations, 2-stock variance, unsystematic risk	statements on risk penalty
Jun'25	**Q5(b) = D23 template**	risk tolerance from utility · index portfolio = systematic risk
Dec'25	–	risk-free + equity mix with SD 24% (CML)
Jun'26	Q5(b) average / expected return, Treynor per stock (8.3)	–

Formula	When to use	SM
HPR = $(P_1 - P_0 +$ dividend$) \div P_0$	Holding-period return; = current yield + capital appreciation.	[SM 299–300]
$E(R_p) = \Sigma W_i R_i$	Portfolio return.	[SM 300]
$\boldsymbol{\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2Cov_{12}}$; Cov = $\rho\sigma_1\sigma_2$	Two-asset risk; three assets: the 3 × 3 matrix.	[SM 301]
Systematic variance = $\beta^2 \sigma_M^2$; residual = total − systematic	Split of total risk (Sharpe single-index model).	[SM 351]
Single-index $\sigma_p^2 = \beta_p^2\sigma_M^2 + \Sigma W_i^2 \times$ residual$_i$	Portfolio variance by the Sharpe index model.	[SM 351]
Risk penalty = risk² ÷ risk tolerance; utility = $E(R_p)$ − penalty	MCQ (vii).	[SM 320]
$S_p = [E(r_p) - r_f] \div \sigma_p$	Optimal risky portfolio maximises this slope (CAL).	[SM 320]

B Portfolio risk · Q5(b)

- **Markowitz** (1952): first scientific approach to risk, return and covariance of a portfolio. [SM 299]
- The expected return of a portfolio is always the weighted average of its holdings' returns; its risk is not (covariance). [SM 301]
- **Systematic risk**: external, market-wide (market, interest-rate, purchasing-power risk) — not diversifiable. **Unsystematic**: the residual = total − systematic; removed by diversification. [SM 301, SM 303]
- An index-like portfolio carries only systematic risk (J25 MCQ). (past MCQ)
- Utility example: return 16%, variance 289, tolerance 60 → penalty 289% ÷ 60 = 4.82%; utility 16% − 4.82% = 11.18%. [SM 320]

PROBLEMS BOX · PORTFOLIO VARIANCE (RECOMPUTED BY SCRIPT m08_patterns.py)

WORKED PATTERN · SM 8.3 ILLUSTRATION 15 (= D23, J25 Q5(b) TEMPLATE)

Share	W	β	Total var.	$\beta^2\sigma_M^2$	Residual
A	0.20	0.40	0.015	0.0016	0.0134
B	0.50	0.50	0.025	0.0025	0.0225
C	0.30	1.10	0.100	0.0121	0.0879

- σ_M = 10%. Portfolio beta 0.08 + 0.25 + 0.33 = **0.66**.
- Sharpe index: $(0.10)^2 \times (0.66)^2$ = 0.0044 + $\Sigma W^2 \times$ residual 0.0140 = **0.0184**.
- Markowitz (Cov AB 0.030, AC 0.020, BC 0.040): **0.0363**. [SM 350–352]

JUN'25 Q5(b) KETON (OUR WORKING)

- Weights 0.35 / 0.25 / 0.40, betas 0.35 / 1.25 / 0.80, σ_M 12%: portfolio β = **0.7550**.
- Residual variances: ASL 0.015 − 0.0018 = **0.0132**; NSL 0.035 − 0.0225 = **0.0125**; ZNL 0.020 − 0.0092 = **0.0108**.
- Markowitz variance (Cov 0.025, 0.060, 0.035) = **0.0334**.

1 · Portfolio beta
Σ weight × beta

↓

2 · Residual variance per share
total − $\beta^2 \times \sigma_M^2$

↓

3 · Sharpe index model
$\beta_p^2\sigma_M^2$ + Σ W^2 × residual

↓

4 · Markowitz
Σ $W^2\sigma^2$ + 2 Σ $W_iW_jCov_{ij}$ (matrix)

PRACTISE THESE (MOST LIKELY FIRST)

D23, J25 Q5(b) · D24 Q5(b), J24 Q5(a), J23 Q5(a) · J26 Q5(b) (8.3) · MCQs D23 Q1(iii), J24 Q1(iii), D24 Q1(ix), J25 Q1(viii), (xv), D25 Q1(vii), PTP Q1(vii) · postal Q4(a), Q7(b) · SM Illus. 1–11, 8.3 Illustrations 1–16 (SM 328–359) · SM exercise MCQs (SM 360–375) · **ICMAI Practice Workbook Ch6 illus. 6, 7, 8, 9, 11, 12** · MCQ bank Q18, Q89–Q95, Q193, Q194, Q196, Q197, Q204.

⚠ **Traps**: residual uses the **market** variance × β^2, not the share's SD; the Sharpe index model ignores the covariances given (they are for Markowitz); square the weights in both models; keep 4 decimals as the question asks.