Module 13 · Maxima and minima P16 · 13.0

SM pp. 662–675 (exercise 676–680). **Q8(a) = a 7-mark profit-maximising sum in 5 of 6 papers** (D24 had a game instead) + MCQs on MR / MC / average cost. Module 13 per paper: **J26-7 · D25-7 · J25-9 · D24-2 · J24-9 · D23-7** (J23-6).

Paper	Q8(a)
Dec'23	dolls: demand x = 1500 − 75p, cost 4x + 1400 → quantity, price, profit
Jun'24	monopolist: cost x^2 + 78x + 300, demand x = (600 − P) ÷ 8
Jun'25	**price discrimination** in two sub-markets (= SM Illus. 7 type)
Dec'25	price and cost as cubic functions of X: maximum-profit X and profit
Jun'26	**maximum profit v maximum revenue** — do they coincide?

1 · Price from the demand function
p in terms of x

↓

2 · Profit π = TR − TC
TR = p × x

↓

3 · dπ/dx = 0 (or MR = MC)
critical values

↓

4 · $d^2\pi/dx^2 < 0$
maximum → price, profit

Rule	Notes	SM
Necessary: dy/dx = 0; sufficient: $d^2y/dx^2 < 0$ maximum, > 0 minimum	Second derivative = 0: point of inflexion.	[SM 663]
Profit maximum where MR = MC and the slope of MR < slope of MC	Same answer as dπ/dx = 0.	[SM 669]
Price discrimination: MC = MR_1 = MR_2; multi-plant: MR = MC_1 = MC_2	Each market priced from its own demand function.	[SM 669]
Minimum average cost: d(C ÷ x)/dx = 0	MCQ (J25); MC from AC: TC = AC × x, then differentiate (D24).	[SM 664]

PROBLEMS BOX · Q8(a) (RECOMPUTED BY SCRIPT m13_patterns.py)

WORKED PATTERNS · SM ILLUSTRATIONS 5 AND 7

- Illus. 5: C = $x^3/3 - 7x^2 + 111x + 50$, x = 100 − p → (x − 1)(x − 11) = 0; maximum at **x = 11**, price **₹89**, profit **₹111.33**. [SM 669–670]
- Illus. 7 (= J25 type): $MR_1 = 80 - 10x_1$, $MR_2 = 180 - 40x_2$, MC = 20 → $x_1 = 6$, $x_2 = 4$; prices **₹50 and ₹100**; profit **₹480**. [SM 671–672]
- Illus. 1: revenue maximum at p = **₹46.11**; Illus. 2: average cost minimum at **x = 1,200**. [SM 664]

OUR WORKINGS

- **J26 Ajooba**: p = 900 − 3q. Revenue maximum q = 150 (₹67,500); profit maximum: 576 − 6q = 0 → **q = 96, p = ₹612, profit ₹26,648** (at q = 150 profit is only ₹17,900) → **they do not coincide**.
- **D25 Suton**: π = $2X^3 - 63X^2 + 648X$ → X = 9 or 12; second derivative −18 at 9 → **sell 9, profit ₹2,187**.
- **J25 Rolex**: 400 − $4Q_1$ = 300 − $4Q_2$ = 60 → Q_1 85 (₹230), Q_2 60 (₹180); profit **₹18,650**.
- **J24 Zuzoo**: π = $522x - 9x^2 - 300$ → **29 sets**, price **₹368**, total cost ₹3,403, profit **₹7,269**.
- **D23 Zoxin**: π = $16x - x^2/75 - 1{,}400$ → **600 dolls**, price **₹12**, profit **₹3,400**.

PRACTISE THESE

J26, D25, J25, J24, D23 Q8(a) · MQP / PTP Q8(a) · SM Illus. 1–8 (SM 663–675), exercise sums 1–5 (SM 680) · MCQs J24 Q1(xii), D24 Q1(vii), J25 Q1(viii) · **ICMAI Practice Workbook Ch8 illus. 1–7** · MCQ bank Q156–Q163, Q252.

Traps: write p from the demand function before TR; check the second derivative (a cubic has a minimum too); profit at the optimum uses the full cost incl. fixed cost.